

DETERMINING THE TEMPERATURE FIELD AND THE  
MELTING RATE OF AN ELECTRODE IN THE MULTIPHASE  
SYSTEM OF THE ELECTRIC-SLAG MELTING PROCESS

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UDC 536.248.2

The problem of melting metals by the electric-slag method is formulated mathematically and some numerical results are shown. The parameters which need to be determined include the rate of electrode melting as well as the melting and crystallization isotherms of the electrode and the ingot.

1. We consider the following metal melting system in the electric slag process (Fig. 1).

At first, the water-cooled cylindrical crystallizer contains an initial layer of metal with the height  $l$  and a layer of liquid slag with the height  $H$  and the electrical resistivity  $\rho$ . A cylindrical metallic electrode is immersed in the slag bath to a certain depth  $h$ , and the voltage supply is switched on. The electric current flowing through the slag bath heats it up and the electrode begins to melt. The molten metal passes through the slag bath, is purged of impurities, settles on top of the initial solid metal layer, and crystallizes. As the crystallizing ingot builds up, it displaces slag in the bath. The depth  $h$  of the electrode in the bath is maintained constant throughout the process by means of an automatic regulator which pulls the electrode down at a speed equal to the melting rate. The purpose of the analysis presented here is to determine the melting rate of the electrode, the temperature fields in the electrode, in the slag bath, and in the ingot, and also the position as well as the shape of crystallization and melting isotherms. A knowledge of these parameters allows one to exert considerable control over the ingot crystallization process for purposes of quality improvement.

In the course of solving the problem, its mathematical formulation was continually refined so as to approach the conditions of a real process\*.

In the following will be given a mathematical description of the heat flow problem and some results of numerical computations. The methods of numerical solution and the evaluation of computed results will be given a detailed treatment in a separate article.

The problem is analyzed with the following stipulations:

- 1) heat travels in the solid and in the liquid phase by conduction, convection in the liquid phase is accounted for only by introducing equivalent thermal conductivities;
- 2) the electrical resistivity of the slag bath does not depend on the temperature, the problem of determining the heat source potential and density fields in the slag is formulated independently of the heat flow problem (see 2);
- 3) the temperature at which drops of molten metal break away from the electrode (the superheat temperature) is considered known;

\*The problem formulation and the boundary constraints were refined with the assistance of L. A. Volokhon-skii, A. A. Nikulin, A. L. Tsikerman, et al., staff members of the Vacuum-Arc Melting Laboratory at the VNIÉTO (All-Union Scientific-Research Institute, Electrical Engineering Department). The authors are also truly grateful to Prof. L. I. Rubinshtein for being always available for consultations and for his valuable advice on formulating the problem.

Computer Center of the P. Stuchki Latvian State University, Riga. Translated from *Inzhenerno-Fizicheski Zhurnal*, Vol. 20, No. 1, pp. 87-95, January, 1971. Original article submitted January 20, 1970.

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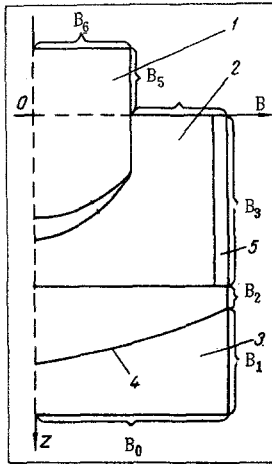


Fig. 1. Schematic representation of an electric-slag mold: 1) melting electrode; 2) slag bath; 3) crystallizing ingot; 4) solid and liquid phase interface; 5) slag crust;  $B_i$  are the various segments of the outer boundary ( $i = 0, 1, 2, 3, 4, 5, 6$ ).

4) the effect of metal drops on the temperature field in the slag bath is disregarded: the temperature change in drops passing through the slag bath is estimated approximately.

In this problem we are dealing with cylindrical symmetry. The cylindrical coordinates  $(r, z)$  are set up as follows: the origin of coordinates lies on the axis of the cylinders, on the top surface of the slag bath; the  $z$ -axis along the axis of the cylinders points in the positive direction downward into the crystallizing ingot (Fig. 1).

Let  $v(t)$  be the linear melting rate of the electrode. In the chosen system of coordinates the electrode then moves at the speed  $v(t)$  and the equation of heat conduction for the electrode will be

$$\operatorname{div} (k_0(T) \operatorname{grad} T) = c_0 \rho_0 \left( \frac{\partial T}{\partial t} + v(t) \frac{\partial T}{\partial z} \right), \quad (1)$$

where  $k_0$  is the thermal conductivity,  $c_0$  is the specific heat, and  $\rho_0$  is the density. The slag bath is stationary in the chosen system of coordinates, it contains heat sources of density  $g(r, z)$ , and the equation of heat conduction for points in it is

$$\operatorname{div} (k_1(T) \operatorname{grad} T) = c_1 \rho_1 \frac{\partial T}{\partial t} - g(r, z). \quad (2)$$

In the chosen system of coordinates the crystallizing ingot moves at the speed  $(R_2^2/R^2)v(t)$ , where  $R_2$  is the electrode radius and  $R$  is the slag bath radius, and the equation of heat conduction for it is

$$\operatorname{div} (k_0(T) \operatorname{grad} T) = c_0 \rho_0 \left( \frac{\partial T}{\partial t} + \frac{R_2^2}{R^2} v(t) \frac{\partial T}{\partial z} \right). \quad (3)$$

We denote by  $z = z_2(r, t)$  the liquid-solid phase interface in the melting ingot. In the chosen system of coordinates the condition for giving off the latent heat of crystallization (the Stefan condition) at this interface can be written as

$$\gamma \rho_0 \left( \frac{\partial z_2}{\partial t} - \frac{R_2^2}{R^2} v(t) \right) = k_{0,\text{sol}} \frac{\partial T}{\partial n} \Big|_{z=z_2-0} - k_{0,\text{liq}} \frac{\partial T}{\partial n} \Big|_{z=z_2+0}, \quad (4)$$

where  $\gamma$  is the specific latent heat of melting for a given metal and  $k_{0,\text{sol}}$ ,  $k_{0,\text{liq}}$  are the thermal conductivities of its solid and of its liquid phase respectively.

We will now derive the Stefan condition for the electrode and slag bath interface. Under actual circumstances, at the electrode which is melting down there forms a small drop of liquid. Disregarding the geometrical dimensions of such a drop, we derive that condition for the electrode boundary in an approximate form. Let  $z = z_0(r, t)$  be the equation of the electrode boundary and  $z = z_1(r, t)$  be the equation of the drop boundary. Then, if  $\theta_0$  is the melting temperature of the metal,  $T|_{z=z_0(r, t)} = \theta_0$ . Let us assume that the drop breaks away at a known superheat temperature  $\theta_1$ , i. e., that  $T|_{z=z_1(r, t)} = \theta_1$ . Since the depth of the electrode immersion in the liquid bath is maintained constant, hence the temperatures and the heat fluxes become as usually equal at the drop boundary:

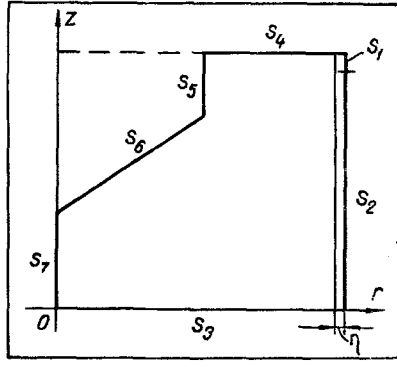


Fig. 2. Referring to the problem of determining the potential field;  $s_i$  are the various outer boundary segments of the region ( $i = 1, 2, 3, 4, 5, 6, 7$ ).

$$k_1 \frac{\partial T}{\partial n} \Big|_{z=z_1+0} = k_{\text{oliq}} \frac{\partial T}{\partial n} \Big|_{z=z_1-0} \quad (5)$$

The Stefan condition is satisfied at the boundary  $z = z_0(r, t)$ :

$$\gamma \rho_0 \left( \frac{\partial z_0(r, t)}{\partial t} - v(t) \right) = k_{\text{osol}} \frac{\partial T}{\partial n} \Big|_{z=z_0-0} - k_{\text{oliq}} \frac{\partial T}{\partial n} \Big|_{z=z_0+0} \quad (6)$$

Integrating equation (1) over region V occupied by the liquid drop, we have

$$\iiint_V \text{div} (k_{\text{oliq}} \text{grad} T) dv = c_{\text{oliq}} \rho_{\text{oliq}} \left( \iiint_V \frac{\partial T}{\partial t} dv + v(t) \iiint_V \frac{\partial T}{\partial z} dv \right) \quad (7)$$

Performing an Ostrogradskii transformation on the left-hand side of Eq. (7) and using relations (5), (6) will yield

$$\begin{aligned} \iiint_V \text{div} (k_{\text{oliq}} \text{grad} T) dv &= \iint_{D_1} k_{\text{oliq}} \frac{\partial T}{\partial n} ds + \iint_{D_2} k_{\text{oliq}} \frac{\partial T}{\partial n} ds \\ &= - \iint_{D_1} \left( k_{\text{oliq}} \frac{\partial T}{\partial n} - \gamma \rho_0 \frac{\partial z_0(r, t)}{\partial t} + \gamma \rho_0 v(t) \right) ds + \iint_{D_2} k_1 \frac{\partial T}{\partial n} ds. \end{aligned} \quad (8)$$

Here  $D_1$  and  $D_2$  are surfaces bounding the liquid drop. The value of  $\partial T / \partial z$  in the right-hand side of Eq. (7) can be approximated by

$$\frac{\partial T}{\partial z} \simeq \frac{\theta_1 - \theta_0}{z_1(r, t) - z_0(r, t)}, \quad (9)$$

and then

$$\iiint_V \frac{\partial T}{\partial z} dv = (\theta_1 - \theta_0) \int_0^{2\pi} d\varphi \int_0^{R_1} \frac{r dr}{z_1(r, t) - z_0(r, t)} \int_{z_0(r, t)}^{z_1(r, t)} dz = (\theta_1 - \theta_0) \pi R_2^2. \quad (10)$$

It is well known that the temperature and the shape of the liquid drop stabilize fast and, therefore, the terms containing  $\partial / \partial t$  may be neglected in (7) and (8). Considering that the magnitude of  $\partial T / \partial n$  varies only slightly along the surfaces  $D_1$ ,  $D_2$  and taking into account relations (8) and (10), we can rewrite (7) as

$$s_2 k_1 \frac{\partial T}{\partial n} \Big|_{z=z_0+0} - s_1 k_0 \frac{\partial T}{\partial n} \Big|_{z=z_0-0} = \gamma \rho_0 v s_1 + c_0 \rho_0 v (\theta_1 - \theta_0) \pi R_2^2, \quad (11)$$

where  $s_1$  and  $s_2$  are the areas of surfaces  $D_1$  and  $D_2$  respectively. Since the volume of a liquid drop is small, we will assume that  $s_1 = s_2 = s$  and that the boundaries of  $D_1$  and  $D_2$  are congruent. Thus, for the electrode boundary  $z = z_0(r, t)$  we finally obtain the condition:

$$k_1 \frac{\partial T}{\partial n} \Big|_{z=z_0+0} - k_0 \frac{\partial T}{\partial n} \Big|_{z=z_0-0} = v \left( \gamma \rho_0 + c_0 \rho_0 (\theta_1 - \theta_0) \frac{\pi R_2^2}{s} \right), \quad (12)$$

and a temperature discontinuity

$$T|_{z=z_0-0} = \theta_0, \quad T|_{z=z_0+0} = \theta_1. \quad (13)$$

We now proceed to the conditions at the inner slag bath and crystallizing ingot interface. We will assume that drops of metal falling off the electrode do instantaneously spread over the surface of the crystallizing ingot and that all the heat they have carried is given off within a region of radius  $R_1 < R$ . At the slag bath and ingot interface  $z = H$ , then, the compatibility condition for heat fluxes and temperatures

$$T|_{z=H+0} = T|_{z=H-0} \text{ and } k_1 \frac{\partial T}{\partial z} = k_0 \frac{\partial T}{\partial z}, \quad (14)$$

will prevail outside that region ( $z = H$ ,  $R_1 < r < R$ ), while the condition

$$k_1 \frac{\partial T}{\partial z} = k_0 \frac{\partial T}{\partial z} + \frac{R_2^2}{R_1^2} v(t) c_0 \rho_0 (\theta_0 - T) \quad (15)$$

and temperature equality will prevail inside that region ( $z = H$ ,  $0 \leq r \leq R_1$ ). The second term on the right-hand side of (15) accounts for the heat carried by drops at temperature  $\theta_2$  into that region.

We next consider the constraints at the outer boundaries of the system. Along segments  $B_0$  and  $B_4$  of the boundary (Fig. 1) we stipulate radiation, according to the Stefan-Boltzmann law, into a medium at a known temperature  $T_i$ :

$$-k \frac{\partial T}{\partial n} = \varepsilon_i \sigma ((T+273)^4 - (T_i+273)^4), \quad (r, z) \in \Gamma_i, \quad i=0, 4, \quad (16)$$

where  $\sigma$  is the Boltzmann constant and  $\varepsilon_0$ ,  $\varepsilon_4$  are the emissivities of the metal and of the liquid bath contents respectively. Along the other boundary segments, where values of the heat transfer coefficients are known from experimental data, the constraints are given as follows:

$$-k \frac{\partial T}{\partial n} = \alpha_i (T - T_i), \quad (r, z) \in \Gamma_i, \quad i=1, 2, 3, 5. \quad (17)$$

Let us examine more closely the constraints at the lateral surfaces of the slag bath and the crystallizing ingot. At the lateral surface of the slag bath, as a result of the cooling action by the external medium, there forms a crust of solid slag along which the crystallizing ingot slides. This crust is thin as compared to the bath dimensions. We will consider its effect approximately, disregarding its geometrical dimensions. For this, we assume that the temperature varies linearly across the crust thickness. The constraint condition can then be easily transferred from outside to inside the crust. If we neglect the Stefan heat at the inside boundary of the solid crust in the slag bath (the position of this boundary stabilizes fast), then we have the continuity conditions for heat fluxes and temperatures inside the crust and, since the temperature distribution in the crust has been assumed linear,

$$k_{\text{sol}} \frac{\partial T}{\partial r} \Big|_{r=R-0} = k_{\text{sol}} \frac{\partial T}{\partial r} \Big|_{r=R-\eta-0}, \quad (18)$$

$$T|_{r=R} = T|_{r=R-\eta-0} + \eta \frac{k_{\text{liq}}}{k_{\text{sol}}} \frac{\partial T}{\partial r} \Big|_{r=R-\eta-0}.$$

At the outer crust surface (boundary  $B_3$ ) the heat is transferred from the surrounding medium in accordance with Newton's law:

$$-k_{\text{sol}} \frac{\partial T}{\partial r} \Big|_{r=R} = \alpha_3 (T - T_3) \Big|_{r=R}. \quad (19)$$

Finally, taking (18) into consideration, the constraint condition (19) on the inside of the crust  $r = R - \eta$  along boundary  $B_3$  becomes

$$-k_{\text{sol}} \left( 1 + \frac{\alpha_3 \eta}{k_{\text{liq}}} \right) \frac{\partial T}{\partial r} = \alpha_3 (T - T_3), \quad (r, z) \in \Gamma_3. \quad (20)$$

Along boundaries  $B_1$  and  $B_2$  we obtain conditions analogous to (20), considering that the solid crust borders on solid metal (boundary  $B_1$ ) and on liquid metal (boundary  $B_2$ ) of the crystallizing ingot:

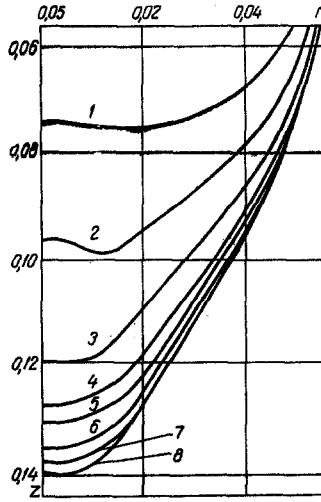


Fig. 3. Changes in the shape of the crystallization isotherm, as a function of time: 1)  $t = 2.25$  min; 2)  $t = 4.5$  min; 3)  $t = 6.75$  min; 4)  $t = 9$  min; 5)  $t = 11.25$  min; 6)  $t = 13.5$  min; 7)  $t = 15.75$  min; 8)  $t = 18$  min;  $r$  and  $z$  in meters.

$$-k_{0,\text{sol}} \left( 1 + \frac{\alpha_1 \eta}{k_1} \right) \frac{\partial T}{\partial r} = \alpha_1 (T - T_1), \quad (r, z) \in \Gamma_1, \quad (21)$$

$$-k_{0,\text{liq}} \left( 1 + \frac{\alpha_2 \eta}{k_1} \right) \frac{\partial T}{\partial r} = \alpha_2 (T - T_2), \quad (r, z) \in \Gamma_2. \quad (22)$$

We will now derive the constraint conditions for the end surface of the electrode (boundary  $B_6$ ). In practice the electrode is sufficiently long to be considered infinite. For calculations by the finite-differences method it is necessary that the space occupied by the electrode be as small as possible. A simple transfer of the constraint condition from infinity, where  $\partial u / \partial n = 0$ , to an electrode section  $z = z_0$  ( $z_0 < 0$ ) may result in a considerable distortion if  $z_0$  is not sufficiently small. Let us derive the relation for this electrode section  $z = z_0$  by taking the conditions of heat transfer into account more precisely. Preliminary calculations have shown that at some distance from the slag bath surface the temperature is almost constant across the electrode section. We are, therefore, allowed to introduce the mean temperature for an ingot section:

$$T_m = \frac{2}{R_2^2} \int_0^{R_2} r T(r, z) dr. \quad (23)$$

Multiplying Eq. (1) for the electrode by  $(2/R_2^2)r$  and integrating with respect to  $r$  from 0 to  $R_2$ , then replacing  $T$  by  $T_m$  at  $r = R$  in the corresponding constraint equation (17), we obtain the following equation for the steady-state process:

$$k_0 \frac{\partial^2 T_m}{\partial z^2} - c_0 \rho_0 v \frac{\partial T_m}{\partial z} + \frac{2}{R_2} \alpha_5 (T_m - T_5) = 0, \quad (24)$$

whose solution (considering that  $T_m$  is bounded at  $z \rightarrow \infty$ ) is

$$T_m = ce^{\gamma_1 z} + T_5, \quad \gamma_1 = \mu + \sqrt{\mu^2 + \beta}, \quad (25)$$

$$\mu = \frac{c_0 \rho_0 v}{k_0}, \quad \beta = \frac{2\alpha_5}{R_2 k_0}, \quad c = \text{const.}$$

The compatibility condition for heat fluxes and temperatures yields for  $T_m$  at  $z = z_0$  (boundary  $B_6$ ) the Newtonian constraint

$$\frac{\partial T_m}{\partial z} = \alpha_6 (T_m - T_5), \quad \alpha_6 = \gamma_1. \quad (26)$$

The initial temperatures at time  $t = 0$  in the electrode ( $u_0$ ), in the slag bath ( $u_1$ ), and in the crystallizing ingot ( $u_2$ ) are given.

2. As has been noted earlier, the electrical resistivity of the slag bath is considered constant. Furthermore, changes in the electrode shape and in the crust thickness have not been taken into account. Under

these assumptions, the problem of determining the heat sources in a slag bath reduces to finding the steady-state potential field  $u$  which satisfies the standard Laplace equation in the slag bath region (Fig. 2). At the electrode boundary ( $s_6$ ) the potential is given as  $u^*$ ; at the slag bath and crystallizing ingot interface (boundary  $s_3$ ) the potential is equal zero; at the free surface of the slag bath (boundary  $s_4$ )  $\partial u/\partial n = 0$ ; on the outside of the solid crust (boundaries  $s_1$  and  $s_2$ ) the potential is given as zero (contact with the electrically conducting crystallizer wall). We will account approximately for the action of a solid crust on the crystallizer wall and on the electrode surface (boundary  $s_5$ ). Since the thickness of the solid crust  $\eta$  is small, the potential drop across the crust thickness will be assumed linear and the conditions on the outside of the crust will be transferred to the inside of the crust, as was done before (see (18), (19), (20)), yielding

$$-\frac{\partial u}{\partial r} = \frac{\rho_{\text{liq}}}{\rho_T \eta} u, \quad (r, z) \in s_2, \quad (27)$$

$$\frac{\partial u}{\partial r} = \frac{\rho_{\text{liq}}}{\rho_S \eta_1} (u - u^*), \quad (r, z) \in s_5, \quad (28)$$

where  $\eta$  and  $\eta_1$  are the thickness of the solid crust at the boundaries  $s_2$  and  $s_5$  respectively. With the potential field  $u$  known, it is easy to determine the density of heat sources in the slag bath:

$$g(r, z) = \frac{1}{\rho} \left\{ \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right\}. \quad (29)$$

3. Without dwelling any further on the numerical method of solving the stated problem and on the necessary transformations, we will only note that this solution was obtained by the Peaceman-Rochford method of variable directions [1], with the Stefan heat at the unknown interface represented in terms of a lumped heat capacity according to O. A. Oleinik - S. L. Kamenomostskaya [2, 3]. We will just point out certain outstanding features of the computation.

The line  $z = H$  (the slag bath and crystallizing ingot interface) participates in the vertical interactions, with conditions (14) and (15) certainly satisfied on it. During horizontal iterations the temperature on this line does not change. The linear velocity  $v(t)$  of the electrode motion begins to be counted when the temperature of the ingot tip immersed in the slag bath reaches the superheat point. The velocity  $v(t)$  is determined from condition (12) at the point ( $r = 0, z = h$ ). At this point one may assume that  $\partial z_0(r, t)/\partial r = 0$ , i. e., that  $\partial T/\partial r = 0$  and, consequently,  $\partial T/\partial n = \partial T/\partial z$ . In the computations it was assumed, approximately, that  $s = \pi R_2^2$ . In this way, the condition from which velocity  $v(t)$  is calculated becomes

$$v(t) = \frac{k_1 \frac{\partial T}{\partial z} \Big|_{z=h+} - k_0 \frac{\partial T}{\partial z} \Big|_{z=h-}}{\gamma \rho_0 + c_0 \rho_0 (\theta_1 - \theta_0)}. \quad (30)$$

In the computations we replace the derivatives  $dT/dz$  by corresponding difference quotients taken from the respective precursory layer. As the electrode acquires a velocity at  $z = h$ , there also occurs a temperature discontinuity along the vertical iterations (see Eq. (13)). If the constraints are nonlinear (heat transfer according to Boltzmann's law, dependence of heat transfer coefficients on the temperature), then on the right-hand sides of the constraint equations we can separate out a linear term whose coefficients are taken from the respective precursory layer. The Peaceman-Rachford method is also used for numerically determining the potential field in the slag bath.

The computation procedure for the problem formulated here was applied to the following variant of an experimental melting process:  $R_1 = 0.015$  m,  $R_2 = 0.3$  m,  $R = 0.055$  m,  $h = 0.029$  m,  $H = 0.0564$  m,  $\eta = 0.0007$  m,  $\eta_1 = 0.0001$  m,  $\rho_{0, \text{sol}} = 7800$  kg/m<sup>3</sup>,  $\rho_{0, \text{liq}} = 7000$  kg/m<sup>3</sup>,  $c_{0, \text{sol}} = 754.2$  J/kg · °C,  $c_{0, \text{liq}} = 838$  J/kg · °C,  $c_1 = 1089$  J/kg · °C,  $k_0 = 34.92$  J/m · sec · °C,  $k_{1, \text{sol}} = 2.33$  J/m · sec · °C,  $k_{1, \text{liq}} = 111.73$  J/m · sec · °C,  $\varepsilon_0 = 0.8$ ,  $\varepsilon_4 = 0.7$ ,  $\theta_0 = 1455$ °C,  $\theta_1 = 1700$ °C,  $u_0 = u_2 = 150$ °C,  $u_1 = 1600$ °C,  $T_0 = T_1 = 100$ °C,  $T_2 = T_3 = 150$ °C,  $T_4 = T_5 = 20$ °C,  $\alpha_2 = \alpha_3 = 2328$  J/m<sup>2</sup> · °C · sec,  $\alpha_1 = 0.154$  T (J/m<sup>2</sup> · °C · sec),

$$\alpha_3 = \begin{cases} 0.047T + 7.65 & \text{for } T \leq 333 \text{ °C;} \\ 0.1T - 10 & \text{for } T > 333 \text{ °C;} \end{cases}$$

$$u^* = 30 \text{ V}, \quad \rho_{\text{liq}} = 0.00324 \text{ } \Omega \cdot \text{m}, \quad \rho_{\text{sol}} = 2 \text{ } \Omega \cdot \text{m}.$$

The temperature field in the slag bath stabilizes relatively fast (approximately within 4 min) and the cavity of liquid metal inside the already crystallized ingot freezes after approximately 18 min. How the shape of the  $T = \theta_0$  isotherm for the crystallizing ingot changes with time is shown in Fig. 3. The melting

$v(t)$  stabilizes approximately after 6.5 min to 2.3 m/h, which agrees closely with experimental data (2.2 m/h). The maximum temperature of the slag bath is 1775°C. The depth of the liquid metal cavity in the steady state differs from the experimentally determined depth by not more than 5%.

Calculations were performed on the BESM-4 computer with a time step of 0.36 sec, an r-step of 0.005 m, and with z-steps chosen as follows: 0.01 m in the electrode (outside the slag bath), 0.0035, 0.0025, and 0.00548 m at various sections of the slag bath, and 0.005 m in the crystallizing ingot.

#### NOTATION

$R_1$	is the radius of the wet region;
$R_2$	is the radius of the electrode;
$R$	is the radius of the ingot;
$h$	is the depth of electrode immersion in the slag bath;
$H$	is the height of the slag bath;
$\eta$	is the thickness of the slag crust at the lateral surface of the bath;
$\eta_1$	is the thickness of the slag crust at the electrode;
$\rho_1, \rho_0, \rho_{\text{sol}}, \rho_0, \rho_{\text{liq}}$	are the densities of the slag, of the solid metal, and of the liquid metal;
$c_1, c_0, c_{\text{sol}}, c_0, c_{\text{liq}}$	are the specific heat of the slag, of the solid metal, and of the liquid metal;
$k_1, k_0, k_{\text{sol}}, k_0, k_{\text{liq}}$	are the thermal conductivities;
$\varepsilon_0, \varepsilon_4$	are the emissivities of the metal and of the slag;
$\theta_0$	is the melting temperature of the metal;
$\theta_1$	is the superheat temperature of the metal;
$\theta_2$	is the temperature of liquid drops after passing through the slag bath;
$u_0, u_1, u_2$	are the initial temperatures of the electrode, of the slag bath, and of the ingot;
$T_0, T_1, \dots, T_5$	are the temperatures of the external media at the boundary segments $\Gamma_0, \Gamma_1, \dots, \Gamma_5$ ;
$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$	are heat transfer coefficients at the boundary segments $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$ ;
$\rho_{\text{liq}}, \rho_{\text{sol}}$	are the electrical resistivities of the liquid and the solid slag;
$u^*$	magnitude of the potential on boundary $s_6$ .

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